5. Symmetry Conditions

A symmetry conditions can be used to reduce the size of a finite element model (or any other computational model). Generally, the symmetry is observed geometrically; that is, the physical domain of interest is symmetric about an axis or plane. Geometric symmetry is not, however, sufficient to ensure that a problem is symmetric. In addition, the boundary conditions and applied loads must be symmetric about the axis or plane of geometric symmetry as well. To illustrate, consider Figure 9a, depicting a thin rectangular plate having a heat source located at the geometric center of the plate. The model is of a heat transfer fin removing heat from a central source (a pipe containing hot fluid, for example) via conduction and convection from the fin. Clearly, the situation depicted is symmetric geometrically. But, is the situation a symmetric problem? The loading is symmetric, since the heat source is centrally located in the domain. We also assume that $k_x = k_y$ so that the material properties are symmetric. Hence, we must examine the boundary conditions to determine if symmetry exists. If, for example, as shown in Figure 9b, the ambient temperatures external to the fin are uniform around the fin and the convection coefficients are the same on all surfaces, the problem is symmetric about both $x$ and $y$ axes and can be solved via the model in Figure 9c.
For this situation, note that the heat from the source is conducted radially and, consequently, across the $x$ axis, the heat flux $q_x$ is zero and, across the $y$ axis, the heat flux $q_y$ must also be zero. These observations reveal the boundary conditions for the quarter-symmetry model shown in Figure 9d and the internal forcing function is taken as $Q/4$. On the other hand, let us assume that the upper edge of the plate is perfectly insulated, as in Figure 9e. In this case, we do not have symmetric conditions about the $x$ axis but symmetry about the $y$ axis exists. For these conditions, we can use the “half-symmetry” model shown in Figure 9f, using the symmetry (boundary) condition $q_x = 0$ across $x = 0$ and apply the internal heat generation term $Q/2$.

Symmetry can be used to reduce the size of finite element models significantly. It must be remembered that symmetry is not simply a geometric occurrence. For symmetry, geometry, loading, material properties, and boundary conditions must all be symmetric (about an axis, axes, or plane) to reduce the model.